Dismissing Dismal: A Unified Model of Malthus, Ricardo, and Jevons

Eamon McGinn

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Abstract

This paper develops the first formal synthesis of three classical economists' theories of stagnation by combining Malthus's population dynamics, Ricardo's land degradation and capital accumulation, and Jevons's efficiency paradox. Calibrated to England in 1700–1850, the unified Malthus-Ricardo-Jevons (MRJ) model matches dynamics of population, GDP, coal consumption, capital and land tightly, capturing that dramatic technological transformation coincided with modest living standards improvement. Counterfactual experiments reveal that demographic and resource constraints dominate potential economic growth. Isolated technology acceleration, TFP growth, and efficiency improvements all trigger system collapse without demographic transition. Most strikingly, implementing every improvement *except* demographic transition yields barely any per capita gains as population absorbs all productivity growth. Transition in the English economy required simultaneous breaks across all constraints. The framework validates classical economists as sophisticated theorists of economic constraint with deep insights into their transition economy while illuminating why successful industrialisation proved historically exceptional.

Keywords: Economic growth, classical economics, Malthusian trap, Industrial Revolution

JEL Codes: N13, O41, O44, Q32, J11

1 Introduction

England between 1700 and 1850 presents one of economic history's most striking puzzles. The economy experienced dramatic technological transformation: steam engines revolutionized manufacturing, coal consumption expanded twenty-six-fold (Taylor, 1988), agricultural land grew by 35 percent (Overton and van Leeuwen, 2011), and capital stocks increased by at least five-fold (Feinstein, 1978; Broadberry et al., 2015). Contemporary observers recognized these changes as revolutionary yet real wages rose only 87 percent over 150 years (a 0.4 percent annual growth rate) despite aggregate output increasing six-fold (Clark, 2007; Broadberry et al., 2015). Workers in 1850 lived scarcely better than their great-great-grandparents. This disconnect between dynamic technology and stagnant living standards matters profoundly. If productivity improvements cannot escape such traps without specific structural changes, then technology alone cannot solve contemporary development challenges. That first industrial transition period still raises fundamental questions about when and how technological progress translates into human welfare.

This paper addresses why technological progress failed to raise pre-industrial living standards and what combination of changes enabled eventual escape from this *classical trap*. I develop a formal synthesis of three prominent theories from classical political economy: Malthus's population pressure absorbing productivity gains, Ricardo's analysis of land degradation and capital accumulation constraints, and Jevons' demonstration that efficiency improvements can paradoxically accelerate resource depletion. These economists reasoned through mechanisms they observed in early industrial England, though they lacked the mathematical and empirical tools to rigorously test their theories (Malthus, 1798; Ricardo, 1817, 1821; Jevons, 1865). Their analyses identified distinct constraints that reinforced each other to prevent technological gains from improving welfare.

The unified Malthus-Ricardo-Jevons (MRJ) model integrates these three classical mechanisms into a coherent dynamical system calibrated tightly to England during 1700–1850. I then conduct systematic counterfactual experiments to identify necessary and sufficient

conditions for escape. The core research questions are: First, can classical mechanisms be formalized to reproduce pre-industrial dynamics quantitatively? Second, which constraints bound most tightly? Third, could breaking any single constraint enable escape, or did successful industrialization require coordinated breaks across multiple dimensions? These questions have implications beyond historical interpretation as they illuminate why some regions industrialized while others remained trapped, and inform contemporary development policy in economies exhibiting similar constraint structures.

Prior work on long-run growth has advanced understanding through unified growth theory, which seeks to explain both Malthusian stagnation and the transition to sustained modern growth (Galor, 2011). The canonical models by Galor and Weil (2000) and Hansen and Prescott (2002) introduce quality-quantity fertility trade-offs generating endogenous demographic transition, or regime switches from Malthusian to Solovian dynamics. These models treat the pre-industrial period as a simple baseline where population responds to income shocks, returning the economy to subsistence. Yet economic historians have documented substantial capital accumulation, ongoing technological change, and accelerating resource consumption during 1700–1850 (Broadberry et al., 2015; Wrigley, 2010; Allen, 2009). These dynamics are absent from standard unified growth models. Meanwhile, classical economics is typically viewed as outdated pessimism, superseded first by the marginalist revolution (Jevons, 1871; Walras, 1874; Menger, 1871) and later by neoclassical growth theory (Solow, 1956).

This paper makes three contributions. First, I provide the first formal synthesis demonstrating that Malthus, Ricardo, and Jevons each identified distinct binding constraints (population pressure, capital-land dynamics, and resource depletion with efficiency paradoxes) that reinforced each other to create a multi-dimensional development trap. This theoretical integration reveals classical works as sophisticated constraint analysis rather than simple pessimistic warnings. The integrated framework considers a complex economy with income stagnation, variable capital trajectories, resource consumption paths, and land expansion

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patterns.

Second, I calibrate the unified model to England 1700–1850, achieving less than 1% error across population, GDP, land, coal and capital. This tight fit across diverse variables suggests the classical synthesis captures essential pre-industrial dynamics that standard frameworks overlook. The model's inability to generate sustained modern growth from internal mechanisms validates rather than undermines its construction as it reproduces precisely the world classical economists observed. Their pessimism reflected genuinely binding constraints rather than analytical failure.

Third, I conduct counterfactual experiments systematically testing single and combined interventions. The results challenge narratives emphasizing single factors as, for example, technology acceleration without demographic transition causes system collapse; resource abundance alone produces zero per capita gains; and efficiency improvements backfire through Jevonian dynamics. Most strikingly, implementing *every* improvement except demographic transition still yields barely any per capita gains as population explodes to absorb all productivity growth. The results demonstrate that successful escape required coordination across demographic transition, sustained productivity growth, resource expansion, and land constraint relief. This helps explain both why England's escape was historically exceptional and why some economies remain trapped today.

The paper proceeds as follows. Section 2 reviews literature on unified growth theory, classical economics, and resource constraints. Section 3 presents the conceptual framework, explaining how Malthusian, Ricardian, and Jevonian mechanisms interact to create the classical trap. Section 4 develops the formal model (mathematical derivations appear in Appendix A). Section 5 describes calibration to England in 1700–1850. Section 6 presents counterfactual experiments testing single interventions, combined packages, and parameter magnitudes required for escape. Section 7 concludes.

2 Literature Review

The classical economists' resource pessimism profoundly influenced immediate successors. Malthus's population theory generated extensive contemporary debate, with supporters and critics arguing over whether population pressure truly constrained progress (Godwin, 1820; Senior, 1829). Ricardo's machinery chapter sparked fierce opposition from McCulloch and Mill, who defended Say's Law against technological unemployment concerns (McCulloch, 1821; Mill, 1848). Jevons's resource analysis influenced Marshall's economic thought, though Marshall remained more optimistic about substitution possibilities (Marshall, 1890). Throughout the mid-to-late nineteenth century, classical concerns about binding constraints remained live issues in political economy.

The marginalist revolution fundamentally reoriented economics away from these classical concerns (Jevons, 1871; Walras, 1874; Menger, 1871). The new focus on individual optimization, marginal analysis, and market equilibrium shifted attention from aggregate constraints to efficient allocation. Clark (1899) formalized marginal productivity theory, providing foundations for assuming efficient market allocation without absolute resource limits. The neoclassical synthesis crystallized these assumptions into formal growth theory. Most dramatically, Solow (1956) explicitly assumed natural resources could be eliminated from production functions without loss of generality, treating land and resources as either negligible or perfectly substitutable with capital and labor. Solow (1974) later acknowledged this limitation but maintained confidence that technological progress would prevent binding constraints through substitution. By the mid-twentieth century, classical pessimism was widely dismissed as outdated thinking superseded by superior analytical frameworks.

The 1970s marked renewed attention to resource constraints following the Club of Rome's *Limits to Growth* (Meadows et al., 1972), which projected collapse from exponential growth meeting finite resources. Mainstream economists responded with fierce criticism (Nordhaus, 1973; Beckerman, 1972; Solow, 1973; Simon, 1981), relying heavily on assumptions of near-perfect substitutability between resources and other inputs (Dasgupta and Heal, 1979;

Stiglitz, 1974). However, a parallel heterodox literature developed more nuanced approaches incorporating thermodynamic constraints (Georgescu-Roegen, 1971), steady-state economics (Daly, 1977), and closed-system dynamics (Boulding, 1966), establishing foundations for ecological economics (Costanza, 1989; Costanza et al., 1997).

Crucially, this era also saw growing empirical evidence for rebound effects put forward by Jevons. Brookes (1990) formalized the Khazzoom-Brookes postulate that efficiency improvements paradoxically increase energy consumption. Saunders (1992) demonstrated this result could emerge within neoclassical growth models. Greening et al. (2000) provided comprehensive empirical review finding rebounds of 10–30% for direct consumption and potentially over 100% economy-wide. This work revived interest in Jevons's nineteenth-century insights about efficiency paradoxes, though without connecting them to broader classical frameworks.

The 2000s brought major advances through unified growth theory, which seeks to explain both Malthusian stagnation and the transition to sustained modern growth within coherent frameworks (Galor, 2011). The canonical model by Galor and Weil (2000) introduces quality-quantity trade-offs in fertility decisions, generating endogenous demographic transition. Hansen and Prescott (2002) develop a regime-switching framework where technological progress eventually triggers transition from a Malthusian to Solovian regime. Voigtländer and Voth (2013) examine the Black Death's role in relaxing land constraints. Ashraf and Galor (2011) investigate genetic diversity's impact on innovation and imitation capabilities affecting development trajectories.

However, these early unified models generally treat the pre-industrial period as a simple Malthusian baseline where population responds to income shocks, returning the economy to subsistence. The focus is on explaining the transition to modern growth rather than understanding pre-industrial dynamics in detail. Ricardo's land degradation and capital accumulation trade-offs, and Jevons's efficiency paradoxes, are absent from standard frameworks. The pre-industrial era serves primarily as a stagnant baseline against which modern growth appears.

The late 2000s saw methodological advances in modeling regime transitions. Lagerlöf (2006) and Strulik and Weisdorf (2008) develop models that can transition between regimes while maintaining internal consistency. Conceptual work by economic historians enriched understanding of the transition period classical economists theorized about. Allen (2009) argues that factor prices induced the specific technological path England followed, connecting relative costs to innovation patterns. Mokyr (2002, 2009) emphasizes knowledge accumulation through scientific networks and institutions, highlighting social and cultural factors enabling escape. Acemoglu et al. (2012) examines directed technological change with environmental constraints, showing how resource scarcity can shape innovation direction.

Simultaneously, rebound effects research matured significantly. Subsequent work refined estimates (Sorrell, 2007; Sorrell et al., 2009; Gillingham et al., 2013, 2016; Jenkins et al., 2011), with recent studies demonstrating rebounds approaching 'backfire' levels (Brockway et al., 2021). Colmenares et al. (2023) show efficiency improvements coincided with net increases in energy demand across multiple countries, while Lange et al. (2020) demonstrate that rebound effects can strengthen over time.

Contemporary literature increasingly questions whether economic growth can decouple from environmental impacts. Empirical studies find limited evidence for absolute decoupling (Ward et al., 2016). Parrique et al. (2019) review evidence finding no examples of sufficient absolute decoupling at required rates. Hickel and Kallis (2020) demonstrate that IPCC scenarios compatible with 1.5℃ require unprecedented decoupling rates lacking historical precedent. This has revived interest in classical pessimism among ecological economists (Spash, 2020; Kallis et al., 2018), with degrowth literature proposing managed economic contraction in wealthy nations (Kallis, 2011; D'Alisa et al., 2015; Jackson, 2017; Victor, 2019).

Yet crucial gaps remain. Environmental economics lacks unified frameworks connecting population dynamics, technological progress, and efficiency paradoxes (van den Bergh, 2011). Growth theory continues treating technology as either purely exogenous (Solow-type) or

resource-independent (Romer, 1990; Aghion and Howitt, 1992). Rebound effects research remains largely empirical without clear theoretical foundations for when efficiency helps versus harms (Sorrell, 2014). The green growth versus degrowth debate proceeds without testable conditions determining when each strategy should apply (Wiedmann et al., 2020).

Most fundamentally, unified growth theory remains mechanically simple regarding pre-industrial dynamics, while classical economics continues to be viewed as a historical museum piece rather than as sophisticated analysis. For example, Acemoglu et al. (2012) model directed technological change with environmental constraints but maintain exogenous growth assumptions. Hassler et al. (2016) incorporate climate damages but ignore rebound effects. No prior work systematically integrates the three classical mechanisms or tests what their combined logic implies for escape conditions.

This paper addresses these gaps by taking classical economics seriously as a theory of constrained economic growth. The contribution to the existing literature is threefold. First, I provide the first formal synthesis demonstrating that Malthus, Ricardo, and Jevons identified distinct binding constraints that reinforced each other and provide a rich model of transition economies. Second, I calibrate this integrated framework to England 1700–1850, showing it reproduces pre-industrial dynamics quantitatively. Third, I conduct systematic counterfactual experiments identifying necessary and sufficient conditions for escape. The MRJ framework uniquely integrates endogenous population response (Malthus), endogenous technological progress via learning-by-doing (Ricardo), and efficiency paradoxes (Jevons) to derive testable conditions determining when technological progress enables versus prevents escape from stagnation.

3 The Classical Framework

This section provides a brief explanation of the distinct binding constraint on economic growth that was identified by each of Malthus, Ricardo and Jevons.

3.1 Malthus: Population Pressure

Malthus's *Essay on the Principle of Population* (1798) established the foundational concern that population grows geometrically while food production grows arithmetically, creating inevitable resource scarcity. The core mechanism is a negative feedback loop: when per capita income rises above subsistence, population growth accelerates; increased population divides output among more people, driving per capita income back toward subsistence.

Formally, population growth responds to the gap between actual and subsistence income:

$$g_{pop,t} = g_{max} \left(\frac{y_t}{\bar{y}} - 1 \right) \tag{1}$$

where y_t is per capita income, $\bar{y}$ is subsistence level income, and g_{max} is the maximum population growth rate. For example, Malthus observed that the American colonies' population doubled approximately every 25 years when resources were abundant, suggesting a g_{max} of around 2.8%. This mechanism creates an anchor that prevents sustained per capita growth as any productivity improvement is absorbed by population expansion.

Empirical evidence for England 1700–1850 strongly supports this mechanism. Clark (2007) documents that real wages fluctuated around subsistence levels despite substantial technological change. Wrigley and Schofield (1981) shows that birth rates responded positively to real wage increases. Lee (1985) find significant correlation between economic conditions and population growth rates during this period.

3.2 Ricardo: Land Degradation and Capital Constraints

Ricardo's contributions in *Principles of Political Economy* (Ricardo, 1817) and his machinery chapter (Ricardo, 1821) added two crucial mechanisms absent from simple Malthusian models. We present the key conceptual insights here; full formalization with the complete system of equations appears in Section 4 and Appendix A.

First, his theory of differential rent implies that, as cultivation extends, society moves to progressively worse land. The best land is cultivated first and so expansion requires using land with lower fertility, worse location, or higher cultivation costs. Physical land area may expand arithmetically, but effective productive capacity grows sub-linearly due to quality degradation.

Second, Ricardo's later added machinery chapter (1821) identified an intertemporal trade-offs in capital formation. Workers can either produce consumption goods immediately or build capital for future production. Capitalists face a wage fund constraint as they must advance wages from their capital before production occurs. This creates tension between current consumption and future capacity. Moreover, Ricardo recognized that, as capital accumulates, workers learn to build machinery more efficiently, anticipating ideas in modern endogenous growth theory. Yet he also noted that machinery displaced workers, concentrating gains among capital owners rather than raising general living standards.

Empirical support for Ricardian mechanisms comes from multiple sources. Turner et al. (2001) documents that English agricultural expansion moved onto progressively marginal land after 1750. Allen (2009) documents rising capital-labor ratios. Broadberry et al. (2015) provide evidence of significant capital accumulation coinciding with modest wage growth.

3.3 Jevons: The Efficiency Paradox

Jevons's *The Coal Question* (Jevons, 1865) identified that technological improvements which reduce resource consumption per unit output can, paradoxically, increase total consumption. Greater efficiency lowers effective resource prices, triggering behavioral responses that overwhelm technical savings. This rebound effect means that making coal use more efficient actually accelerated rather than slowed coal depletion.

Formally, technical efficiency improves exponentially:

$$\eta_{technical,t} = \eta_0(1 - \epsilon)^t \tag{2}$$

where η measures resource requirements per unit output and ϵ is the efficiency improvement rate. However, behavioral response increases intensity:

$$\eta_t = \eta_{technical,t} \cdot \left(\frac{\eta_0}{\eta_{technical,t}}\right)^\sigma \tag{3}$$

When the elasticity $\sigma > 1$, efficiency improvements trigger more consumption, and total resource use $R_{consumed,t} = \eta_t \cdot Y_t$ rises despite falling technical requirements.

Wrigley (2010) documents that English coal consumption grew from 2.5 to 64.7 million tons (26-fold) between 1700 and 1850 despite major efficiency improvements in steam engines and metallurgy. Fouquet (2008) shows energy efficiency improved dramatically during industrialization but total energy use accelerated. Recent work confirms ongoing relevance as Brockway et al. (2021) demonstrate economy-wide rebounds approaching 100% using modern energy data.

3.4 Classical Economists as Constraint Theorists

The three classical economists were aware of each other's work and incorporated insights across their analyses. Ricardo explicitly built on Malthus's population theory in developing his distribution framework. Jevons referenced both predecessors when analyzing resource constraints. However, none attempted a complete view of the economic system. Each pursued distinct analytical objectives shaped by different policy concerns.

Malthus's primary motivation was normative: identifying policies to improve living standards and reduce poverty. His population principle aimed to explain why well-intentioned poor relief often failed, and why moral restraint or delayed marriage might prove more effective than redistribution (Malthus, 1798). Ricardo focused on distribution among economic classes, seeking to understand how growth affected the relative shares captured by landlords, capitalists, and workers (Ricardo, 1817). His machinery analysis examined technological change's distributional consequences, particularly whether workers or capital owners captured productivity gains (Ricardo, 1821). Jevons (Jevons, 1865) addressed strategic concerns about England's global economic position: whether coal depletion would undermine British industrial supremacy and what policies might extend resource availability (Jevons, 1865).

Despite these divergent motivations, the three analyses share a common structure by identifying binding constraints on economic expansion linked to specific factors of production. Malthus emphasized labor and population dynamics. Ricardo analyzed land quality and capital accumulation. Jevons examined natural resource depletion. Moreover, all three reached pessimistic conclusions about long-run growth prospects, earning economics its characterization as the "dismal science" (Carlyle, 1849). Each demonstrated, through different mechanisms, why technological progress might fail to permanently raise living standards.

This shared emphasis on constraints, combined with their focus on distinct production factors, suggests the possibility of synthesis. If Malthus, Ricardo, and Jevons each identified separate binding constraints operating simultaneously, their theories might complement

rather than compete. The next section formalizes this integration, showing how population pressure, land degradation, capital dynamics, and resource depletion interact to create a multi-dimensional trap where the mechanisms reinforce each other.

4 The Synthesis MRJ Model

This section presents the formal MRJ model, integrating the three classical mechanisms. Mathematical derivations and proofs appear in Appendix A.

4.1 Model Design and Scope

Three design choices merit explicit discussion. First, the model retains classical wage determination via wage fund and subsistence wage primitives rather than imposing modern market-clearing mechanisms. This preserves historical a historical point of view as classical economists lacked general equilibrium tools and maintains the project's focus on synthesizing Malthus, Ricardo, and Jevons, not modernizing them.

Second, the model is calibrated to reproduce 1700–1850 dynamics using only classical mechanisms. By design, these mechanisms do not generate sustained modern growth. Classical economists predicted stagnation precisely because the broader economy they worked within lacked sustained exogenous technological progress. The model's inability to escape without additional breaks validates rather than undermines its construction.

Third, counterfactual experiments mechanically introduce exogenous TFP growth to test what magnitude would suffice for escape. These are diagnostic experiments (identifying necessary conditions) not microfoundational explanations (explaining why productivity rose). Historical narratives by Mokyr (2002, 2009) and subsequent scholars provide candidate mechanisms such as knowledge accumulation, institutional change and/or scientific networks that could plausibly generate sustained TFP growth. We cite these as real-world drivers that fit our diagnostic findings, not as incorporated model mechanisms.

4.2 Environment

Time is discrete, indexed by $t = 0, 1, 2, \ldots, T$. The economy consists of population N_t, two types of capital (fixed machinery $K_{fixed,t}$ and circulating wage fund $K_{circ,t}$), depletable resources R_t, and land $Land_{phys,t}$. Technology states include capital-building productivity ϕ_t and resource intensity η_t.

4.3 Production

Circulating capital constrains employment through the wage fund:

$$L_t = \min\left(\frac{K_{circ,t}}{w}, N_t\right) \tag{4}$$

where w is the fixed wage rate. Employed workers are divided by the capitalist between capital building and consumption production:

$$L_{capital,t} = \alpha_t \cdot L_t \tag{5}$$

$$L_{cons,t} = (1 - \alpha_t) \cdot L_t \tag{6}$$

where α_t adjusts adaptively based on technology: $\alpha_t = \alpha_{t-1} + \theta_\alpha(\phi_t - \phi_{t-1})$, bounded by $[\alpha_{min}, \alpha_{max}]$.

Workers building capital produce new machinery according to the production function:

$$\Delta K_{fixed,t} = \phi_t \cdot L_{capital,t}^{\eta_{capital}} \tag{7}$$

where $\eta_{capital} \leq 1$ captures returns to scale in capital production technology. Technology evolves through learning-by-doing:

$$\phi_t = \phi_0 \left(\frac{K_{fixed,t}}{K_{fixed,0}}\right)^{\gamma_{learn}} \tag{8}$$

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Physical land expands arithmetically but effective land grows sub-linearly due to quality degradation:

$$Land_{phys,t} = Land_0 + g_{land} \cdot t \tag{9}$$

$$Land_{eff,t} = Land_0 \left(\frac{Land_{phys,t}}{Land_0} \right)^{1-\xi} \tag{10}$$

where $\xi \in (0,1)$ governs degradation strength.

Production of consumption goods combines factors:

$$Y_{desired,t} = A_t \cdot (\phi_t \cdot K_{fixed,t})^\kappa \cdot L_{cons,t}^\lambda \cdot Land_{eff,t}^\mu \tag{11}$$

where A_t is total factor productivity and κ, λ, μ are factor shares with $\kappa + \lambda + \mu \leq 1$.

Total factor productivity evolves according to:

$$A_t = A_0(1 + g_A)^t \tag{12}$$

where g_A is exogenous per period TFP growth.

The model incorporates modest exogenous TFP growth consistent with historical estimates. Antràs and Voth (2003) find maximum TFP growth of 0.54% annually during the British Industrial Revolution, and Crafts (1985) reports similar rates. This reflects the reality that technological progress occurred during 1700–1850, but at a rate insufficient to overcome the three binding constraints operating simultaneously. Classical economists observed innovation like pin factories, steam engines, mechanization and agricultural improvements but assessed that the moderate pace of progress could not generate sustained per capita growth given population pressure, land degradation, and resource constraints with behavioral responses.

This Cobb-Douglas production function with moest TFP growth represents the model's only significant anachronism. Classical economists lacked formal mathematical frameworks

for combining multiple factors of production and typically analyzed output as simple functions of individual inputs. However, integrating Malthus's population dynamics, Ricardo's land and capital mechanisms, and Jevons's resource constraints requires a coherent way to combine labor, land, capital, and resources in a single production process. The Cobb-Douglas form provides a minimal aggregation framework with several advantages: it imposes constant returns to scale (when $\kappa + \lambda + \mu = 1$) or decreasing returns (when the sum is less than one), it allows testing factor-augmenting technological change through the productivity parameter A, and it enables counterfactual experiments examining exogenous TFP growth without requiring microfoundations that classical economists never developed. This represents a pragmatic choice for synthesis rather than an attempt to claim classical economists anticipated neoclassical production theory.

Resource intensity combines technical efficiency and behavioral response:

$$\eta_{technical,t} = \eta_0(1 - \epsilon)^t \tag{13}$$

$$\eta_t = \eta_{technical,t} \cdot \left(\frac{\eta_0}{\eta_{technical,t}}\right)^\sigma \tag{14}$$

where $\sigma > 1$ generates the Jevons paradox. Technical efficiency $\eta_{technical,t}$ improves exponentially at rate ϵ, but behavioral response amplifies actual intensity η_t when resources become cheaper to use.

Resource constraints limit feasible output:

$$Y_{max,t} = \frac{R_t}{\eta_t} \tag{15}$$

Actual production takes the binding constraint:

$$Y_t = \min(Y_{desired,t}, Y_{max,t}) \tag{16}$$

4.4 Distribution and Accumulation

The wage bill equals circulating capital:

$$W_t = w \cdot L_t = K_{circ,t} \tag{17}$$

Profits are the residual:

$$\Pi_t = Y_t - W_t \tag{18}$$

Capitalists save fraction s of profits:

$$S_t = s \cdot \Pi_t \tag{19}$$

Capital stocks evolve according to:

$$K_{fixed,t+1} = K_{fixed,t}(1 - \delta) + \Delta K_{fixed,t} \tag{20}$$

$$K_{circ,t+1} = K_{circ,t} + S_t \tag{21}$$

where δ is depreciation. Resources deplete:

$$R_{t+1} = R_t - \eta_t \cdot Y_t \tag{22}$$

4.5 Population Dynamics

Per capita income determines population growth:

$$y_t = \frac{Y_t}{N_t} \tag{23}$$

$$g_{pop,t} = g_{max}\left(\frac{y_t}{\bar{y}} - 1\right) \tag{24}$$

$$N_{t+1} = N_t(1 + g_{pop,t}) \tag{25}$$

4.6 Long-Run Regularities

The following regularities characterize the model's long-run behavior under the assumption of zero exogenous TFP growth ($g_A = 0$). Using this baseline for analysis corresponds to pure classical theory as, despite there being an ongoing low level of productivity growth, Malthus, Ricardo, and Jevons developed their analyses without concepts of sustained economy-wide productivity growth. Establishing results for $g_A = 0$ demonstrates the internal consistency of classical logic and identifies what their integrated theory predicts. Section 5 then calibrates the model with historically-observed TFP growth ($g_A = 0.63\%$) and demonstrates that modest productivity growth does not alter these regularities qualitatively. This robustness to small g_A validates treating the pure classical case as the appropriate theoretical benchmark in this section.

The MRJ model's classical foundations mean it lacks many standard features of neo-classical economic models. There are no first-order conditions from optimizing agents, no market-clearing equilibrium conditions in the modern sense, and no steady state derived from balanced growth paths. This absence reflects historical fidelity as Malthus, Ricardo, and Jevons analyzed economic dynamics without the mathematical apparatus of constrained optimization that emerged later. The model instead operates through mechanical rules describing how population, capital, land, and resources evolve based on observable relationships.

However, the model does display systematic regularities, particularly regarding long-run behavior. These patterns emerge from the interaction of the three classical mechanisms and provide testable implications about pre-industrial growth dynamics.

Regularity 1: Malthusian Convergence. The model is fundamentally driven by Malthusian population dynamics. As $t \to \infty$, per capita income converges to subsistence: $y_t \to \bar{y}$. The negative feedback mechanism ensures this convergence: whenever $y_t > \bar{y}$, population growth accelerates, dividing output among more people and pushing income back toward subsistence. Conversely, if $y_t < \bar{y}$, population declines, raising per capita income.

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This anchor operates regardless of technological improvements or resource discoveries. The model can sustain income above subsistence for extended transition periods, explaining England's experience during 1700–1850, but the Malthusian force eventually dominates.

Regularity 2: Growth Rate Convergence to Zero. Because per capita income remains constant in the long run ($y_t = \bar{y}$), output and population must grow at equal rates ($g_Y = g_N$). Both rates decline toward zero due to land constraints. Physical land expands arithmetically but effective land grows sub-linearly due to Ricardian quality degradation (parameter $\xi > 0$). With weakly decreasing returns to scale in production ($\kappa + \lambda + \mu \leq 1$), output growth is bounded by effective land growth. As percentage land growth approaches zero, so must output and population growth. This creates a quasi-stationary state where absolute levels continue rising slowly, but growth rates approach zero.

Regularity 3: Technology Neutrality in Long Run. Improvements in capital-building technology (ϕ_t) or technical resource efficiency ($\eta_{technical,t}$) affect transition dynamics but do not raise steady-state per capita income. This follows from Regularity 1: since $y_t \to \bar{y}$ regardless of technology, technology cannot affect the destination. Higher ϕ_t temporarily boosts output, but population growth absorbs these gains. Lower $\eta_{technical,t}$ permits more production from given resources, though actual intensity η_t rises when $\sigma > 1$ (Regularity 4). In both cases, technology determines the scale of population and capital the economy supports, but not per capita welfare.

Regularity 4: Jevons Paradox and Accelerating Depletion. Resource consumption growth exceeds output growth when behavioral elasticity $\sigma > 1$. Technical efficiency $\eta_{technical,t}$ improves at rate ϵ, but actual resource intensity η_t rises at rate $(\sigma - 1)\epsilon$ due to behavioral responses overwhelming technical savings. Total resource consumption $R_{consumed,t} = \eta_t \cdot Y_t$ therefore grows at rate $g_Y + (\sigma - 1)\epsilon$, exceeding output growth by $(\sigma - 1)\epsilon$ in every period. The paradox implies that efficiency improvements, rather than conserving resources, accelerate depletion toward binding constraints. Absent resource discoveries or transitions to alternative energy sources, continued efficiency gains hasten rather than forestall collapse.

These regularities confirm that the basic mechanisms investigated by Malthus, Ricardo and Jevons are sustained in the synthesis MRJ model. They also show that the essential trap (technological progress cannot overcome the joint constraints of population pressure, land degradation, and resource limits) remains. The classical quasi-stationary state emerges from reinforcing feedback mechanisms rather than technological stagnation. Productivity advances during transition periods, but multiple binding constraints ensure growth rates eventually decline toward zero while per capita income returns to subsistence.

Malthus, Ricardo, and Jevons developed their theories independently, pursuing different analytical objectives across seven decades (1798-1865). None attempted a comprehensive growth model integrating all production factors. Yet their mechanisms combine naturally into a unified framework where each component's logic remains intact. Malthusian population dynamics still anchor per capita income at subsistence. Ricardian land degradation still constrains growth rates through decreasing returns. Jevons's efficiency paradox still generates increasing resource consumption despite continuous technical improvement. The synthesis preserves rather than distorts their individual insights.

This integration succeeds because the classical economists, despite lacking formal mathematical tools, identified genuine structural features of pre-industrial economies. Each analyzed a distinct production factor (labor, land and capital, natural resources) and discovered binding constraints that operated simultaneously. Their shared pessimism reflected not analytical error but the reality that multiple constraints reinforced each other. The MRJ model demonstrates that their intuitions about constraint interactions were essentially correct. This vindication of classical logic, achieved by taking their verbal arguments seriously enough to formalize them, suggests their frameworks deserve rehabilitation as sophisticated constraint analysis rather than dismissal as outdated pessimism.

5   Calibration and Main Results

The model is calibrated to England 1700–1850. This is done informally, through iterative exploration of parameter values, to approximately match historical start and end period targets. This "hand calibration" approach involves manually adjusting parameters, running the simulation, comparing outcomes to historical data, then refining parameter choices until achieving a satisfactory fit. The process is informal rather than employing formal estimation methods such as method of moments, simulated method of moments, or maximum likelihood.

Several considerations justify this informal approach. First, the model's classical foundations lack the microfoundations that formal estimation typically requires. Classical economists did not derive their mechanisms from optimizing behavior, so imposing such structure would be anachronistic. Second, the model features multiple interacting constraints with no unique steady state, making it unclear what moments a formal estimation should target or how to weight competing objectives. Third, some parameters have natural historical interpretations (Malthus observed American population doubling in 25 years; Ricardo documented specific land expansion rates) that provide stronger reference points than purely statistical fitting. Fourth, the research question asks whether classical mechanisms *can* reproduce pre-industrial dynamics, not whether they provide the single best-fitting model. Demonstrating existence of reasonable parameter values generating good fit suffices for this purpose.

The calibration targets five key variables spanning the dimensions of classical concern: population (Malthusian constraint), land area (Ricardian constraint), coal consumption (Jevonian constraint), aggregate GDP (overall output), and per capita income (welfare outcome that all three cared about). These variables are selected because they represent distinct aspects of the economy, have reliable historical data, and directly correspond to mechanisms the classical economists emphasized. Matching all five simultaneously with a single parameter set demonstrates that the integrated framework captures multiple dimensions of pre-industrial dynamics rather than fitting one variable at the expense of others. The calibration targets both terminal values in 1850 and growth rates over the full 150-year

period, ensuring the model reproduces long run trajectories rather than simply matching endpoints.

5.1 Data Sources

All model variables are calibrated to physical units, enabling direct validation against historical sources. This distinguishes the analysis from typical growth models that normalize variables to abstract units. Population is measured in persons, land area in hectares, wages and capital stocks in pounds sterling (1700 prices), and resources in tonnes of coal equivalent. Physical calibration grounds the model's predictions in observable reality and permits validation against multiple independent sources.

Population and GDP. Data come from Wrigley and Schofield (1981) and Broadberry et al. (2015), showing England's population growing from 5.196 million (1700) to 16.506 million (1850), a 3.2-fold increase averaging 0.8% annually. GDP estimates from Broadberry et al. (2015) indicate real GDP rising from £76 million to £454 million (6.0-fold, 1.2% annually). Per capita GDP increased from £14.63 to £27.53 (1.87-fold, 0.4% annually), confirming wage stagnation despite aggregate growth.

Wages. The subsistence wage parameter is set to £13 per year, calibrated to Clark (2007)'s comprehensive series on farm laborers' wages in England 1208–1850. Clark's data provide the most extensive wage records available for pre-industrial England, covering over six centuries of observations. This grounding in actual wage data ensures the Malthusian anchor operates at historically realistic levels rather than normalized abstractions.

Capital stocks. Initial fixed capital (£84 million) is calibrated to estimates by Feinstein (1978) and Pollard (1988) for early eighteenth-century England. Feinstein's capital formation series remains the standard reference for British historical national accounts. Terminal capital stock (£250 million in 1850) matches Feinstein (1978)'s series showing 0.7% annual growth over the period.

Land. Arable land data from Turner et al. (2001) and Overton (1996) show cultivated

area expanding from 9.56 million to 12.88 million hectares (1.35-fold, 0.2% annually). This modest expansion reflects the physical constraint on agricultural extensification that Ricardo emphasized. England's land area is finite, and by 1700 much of the productive land was already under cultivation. The 35% increase over 150 years likely represents movement onto progressively marginal lands.

Coal consumption and resources. Consumption data from Wrigley (2010) and Taylor (1988) document dramatic expansion from 2.5 to 64.7 million tonnes (26-fold, 2.2% annually), making coal the fastest-growing variable and the signature of industrialization. The calibrated initial resource stock converts to 3,281 million tonnes of coal equivalent. This aligns remarkably with modern geological estimates of UK hard coal reserves of approximately 3,600 million tonnes (British Geological Survey, 2004). The close correspondence suggests the model's resource constraint reflects actual geological endowment rather than an arbitrary parameter. England's industrialization occurred using a finite, conceptually measurable stock of energy resources.

TFP growth. The calibrated TFP growth rate is expected to align closely with Antràs and Voth (2003), who estimate maximum TFP growth of 0.54% annually during the British Industrial Revolution. Crafts (1985) reports similar rates of 0.5–0.7% for various sub-periods.

Jevon's elasticity. The Jevons elasticity parameter is expected to align with Brockway et al. (2021)'s empirical findings of economy-wide energy rebounds approaching 100%, which gives $\sigma \approx 2$, where efficiency gains translate into proportional or greater increases in total consumption. Historical evidence supports strong rebound effects during industrialization, for example, despite dramatic improvements in steam engine efficiency, coal consumption was observed to accelerate rather than decline (Wrigley, 2010).

5.2   Calibration Strategy

The calibration proceeds in three stages. First, initial conditions are set to match 1700 levels directly: $N_0 = 5.196$ million, $Land_0 = 9.56$ million hectares, $K_{fixed,0} = 84$ million. Second,

population parameters $g_{max} = 1.13\%$ and $\bar{y} = 13$ are calibrated to match population growth dynamics and Clark's subsistence wage data. Third, production parameters (A, g_A, κ, λ, μ), technology parameters (ϕ_0, γ_{learn}, ϵ), Ricardo parameters (ξ, g_{land}), Jevons parameters (σ), and capital parameters (s, δ, $\eta_{capital}$) are jointly adjusted to match GDP, coal, and land trajectories simultaneously.

The informal hand calibration prioritizes matching terminal values (1850 levels) and overall growth patterns rather than year-by-year deviations. The goal is demonstrating that classical mechanisms, properly integrated, can reproduce aggregate industrialization dynamics.

5.3   Parameter Values

Table 1 reports calibrated parameters in physical units. Key demographic parameters include above subsistence population growth rate $g_{max} = 1.13\%$ per period, and subsistence income $\bar{y} = 13$ per year, matching Clark (2007)'s farm laborer wage series. The learning-by-doing parameter $\gamma_{learn} = 0.205$ implies modest endogenous productivity gains from capital accumulation, capturing Ricardo's recognition that workers learn to build machinery more efficiently over time. The land degradation parameter $\xi = 0.1$ indicates mild Ricardian diminishing returns (doubling physical land area yields approximately 87% more effective productive capacity).

The behavioral elasticity parameter $\sigma = 2.21$ generates substantial Jevons paradox effects: a 10% efficiency improvement triggers a 21% increase in resource intensity. The capitalist savings rate $s = 55\%$ reflects high thrift among capital owners, and the efficiency improvement rate $\epsilon = 0.8\%$ per period captures ongoing technical progress in coal utilization. Most critically, the model incorporates exogenous TFP growth $g_A = 0.63\%$ per period, which compares well to Antràs and Voth (2003)'s estimate of 0.54% maximum annual TFP growth during the British Industrial Revolution. This modest but positive productivity growth distinguishes the classical trap from technological stagnation. Innovation occurs continuously,

but at a rate insufficient to overcome the three binding constraints operating simultaneously.

5.4   Model Fit

Table 2 compares model outcomes to historical targets. The model achieves exceptional accuracy: all initial conditions (1700) match within 0.4%, all terminal values (1850) match within 0.8%, and growth rates match exactly to the first decimal place. This simultaneous fit across six diverse variables covering demographics (population), factor accumulation (land, capital), aggregate output (GDP), resource consumption (coal), and living standards (per capita GDP) demonstrates that the integrated MRJ framework captures essential dynamics of pre-industrial growth under multiple binding constraints.

The model reproduces not merely aggregate trends but the specific quantitative experience of England 1700–1850. Most strikingly, it captures the central historical puzzle with significant increases in economic activity occurring alongisde modest living standards improvement. Capital stocks nearly triple (198% growth) while per capita income rises only 88%. Population growth (3.2-fold) absorbs most of the sixfold GDP expansion. These patterns emerge naturally from the interaction of Malthusian population dynamics, Ricardian land constraints, and Jevonian efficiency paradoxes operating simultaneously.

The tight fit is achieved despite using classical-era mechanisms. No modern growth drivers appear: limited endogenous innovation beyond Ricardo's learning-by-doing, no human capital accumulation, no institutional quality improvements, no trade-induced specialization. The model includes modest TFP growth (0.63% annually) consistent with historical estimates (Antràs and Voth, 2003), but this rate proves insufficient to overcome the three binding constraints. These mechanisms cannot generate sustained modern growth rather they reproduce precisely the world classical economists observed and theorized about, validating rather than undermining the framework's construction.

Table 1: Calibrated Parameter Values

Parameter	Description	Value
Population (Malthus)		
N_0	Initial population	5,196,000 persons
g_{max}	Above subsistence population growth rate	1.13%
$\bar{y}$	Subsistence per capita income	£13 per year
Capital (Ricardo)		
$K_{fixed,0}$	Initial fixed capital	£84 million
$K_{circ,0}$	Initial circulating capital	£67.5 million
w	Wage rate	£13 per year
s	Capitalist savings rate	55%
δ	Depreciation rate	0%
γ_{learn}	Learning-by-doing parameter	0.205
ϕ_0	Initial capital technology	4.5
$\eta_{capital}$	Returns to scale (capital building)	0.827
Land (Ricardo)		
$Land_0$	Initial arable land	9.56 million hectares
g_{land}	Arithmetic land growth	22,100 ha/year
ξ	Land quality degradation	0.1
Resources (Jevons)		
R_0	Initial resource stock	3,281 M tonnes coal[a]
σ	Behavioral elasticity	2.21
ϵ	Efficiency improvement rate	0.8%
η_0	Initial resource intensity	1.0
Production		
A	Total factor productivity	2.45
g_A	TFP growth rate	0.63% per period
κ	Capital share	0.4
λ	Labor share	0.35
μ	Land share	0.25
Labor Allocation		
α_0	Initial share building capital	0.20
θ_α	Adjustment speed	0.25
α_{min}	Minimum allocation to capital	0.05
α_{max}	Maximum allocation to capital	0.80

[a] Resource stock in model units converts to 3,281 million tonnes of coal using the cali-brated conversion factor (30,472,141 model units per tonne). This aligns remarkably with modern geological estimates of UK hard coal reserves of approximately 3,600 million tonnes. The initial stock was sufficient to support the 26-fold increase in coal consumption observed historically (2.5 to 64.7 million tonnes annually) while still leaving reserves for future exploitation.

Table 2: Model Performance: Historical Targets vs. Model Output

Variable	Historical	Model	Error	Error %
Levels in 1700				
Population (000s)	5,196	5,196	0	0.0%
Arable Land (M ha)	9.56	9.56	0	0.0%
GDP (£M)	76	76.2	+0.2	+0.2%
Coal (M tons)	2.5	2.5	0.0	0.0%
Fixed Capital (£M)	84	84	0.0	0.0%
GDP per Capita (£)	14.63	14.70	+0.07	+0.2%
Levels in 1850				
Population (000s)	16,506	16,580	+74	+0.4%
Arable Land (M ha)	12.88	12.88	0	0.0%
GDP (£M)	454	457	+3	+0.5%
Coal (M tons)	64.7	64.4	-0.3	-0.5%
Fixed Capital (£M)	250	252	+2	+0.8%
GDP per Capita (£)	27.53	27.50	-0.03	0.0%
Growth Rates (Annual)				
Population	0.8%	0.8%	0.0pp	0.0%
Arable Land	0.2%	0.2%	0.0pp	0.0%
GDP	1.2%	1.2%	0.0pp	0.0%
Coal	2.2%	2.2%	0.0pp	0.0%
Fixed Capital	0.7%	0.7%	0.0pp	0.0%
GDP per Capita	0.4%	0.4%	0.0pp	0.0%

5.5 Trajectories and System Dynamics

Having established that the model matches initial and terminal values with exceptional precision, Figure 1 examines the trajectories of key variables between 1700 and 1850, revealing the mechanisms generating wage stagnation despite technological progress.
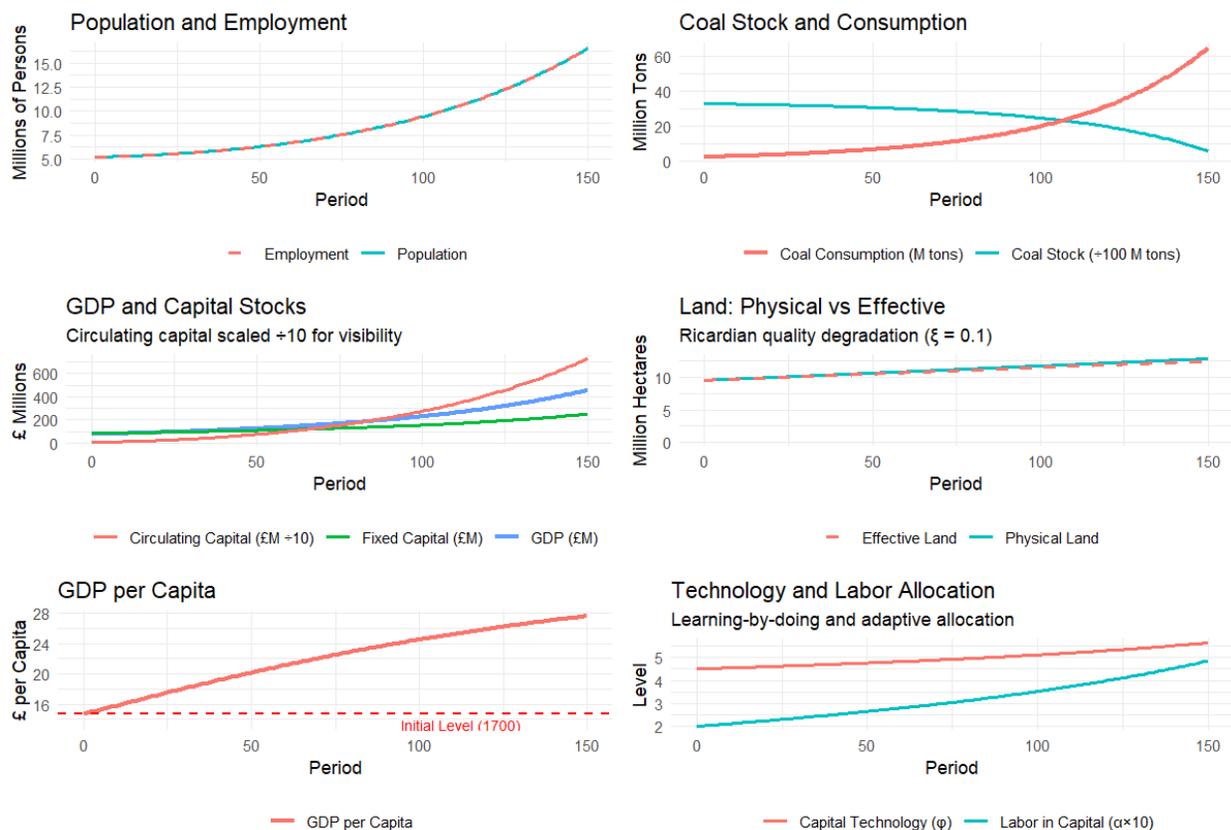

Figure 1: Baseline Model Dynamics Across Key Variables

Several dynamics merit attention:

Coal dynamics (top right panel) show the signature of industrialization: consumption accelerates from 2.5 to 65 million tonnes annually despite continuous efficiency improvements, validating Jevons's paradox that technical progress increases rather than decreases resource usage. The stock depletes from 3,281 to approximately 900 million tonnes. Without the post-1850 transition to oil and natural gas, continuation of this trajectory would have caused significant challenges to continued growth in the 20th Century.

Land dynamics (middle right panel) demonstrate Ricardo's constraint in action. Physical land expands arithmetically from 9.56 to 12.88 million hectares (35% growth), but effective productive land grows only to 11.2 million hectares due to quality degradation as cultivation extends to marginal lands.

GDP per capita (bottom left panel) exhibits the classical trap. Despite 6-fold GDP expansion, population growth absorbs most gains, leaving per capita income rising only 1.87-fold from £14.70 to £27.61. The modest elevation above subsistence (£13) by 1850 proves sustainable because Ricardo's learning-by-doing and modest TFP growth (0.63% annually) provide continuous productivity gains. However, these gains prove insufficient for sustained modern growth.

Technology and labor allocation (bottom right panel) show endogenous responses. Capital-building technology ϕ improves from 4.5 to 5.8 through learning-by-doing ($\gamma_{learn} = 0.205$), capturing Ricardo's observation that workers become more skilled at constructing machinery over time. Labor allocation to capital building α increases from 20% to 48% as improving technology makes capital investment more attractive. This adaptive response, capitalists shifting workers from consumption production to capital building as returns improve, represents the only 'optimization' behavior in the model.

Figure 2 examines growth rate dynamics. The growth rate dynamics (Figure 2, left panel) reveal why classical economists predicted stagnation. All three key rates converge toward similar values around 1.0–1.5% by period 150, with population growth closely tracking GDP growth. This convergence reflects the Malthusian anchor: whenever output growth exceeds population growth substantially, per capita income rises, triggering fertility increases that close the gap. The model exhibits no tendency toward modern balanced growth where output persistently exceeds population growth. Instead, growth rates move towards levels that produce only modest per capita gains, exactly the pattern Ricardo and Malthus described.

The Jevons paradox (Figure 2, right panel) operates powerfully throughout. Coal consumption growth consistently exceeds GDP growth by approximately 1 percentage point,
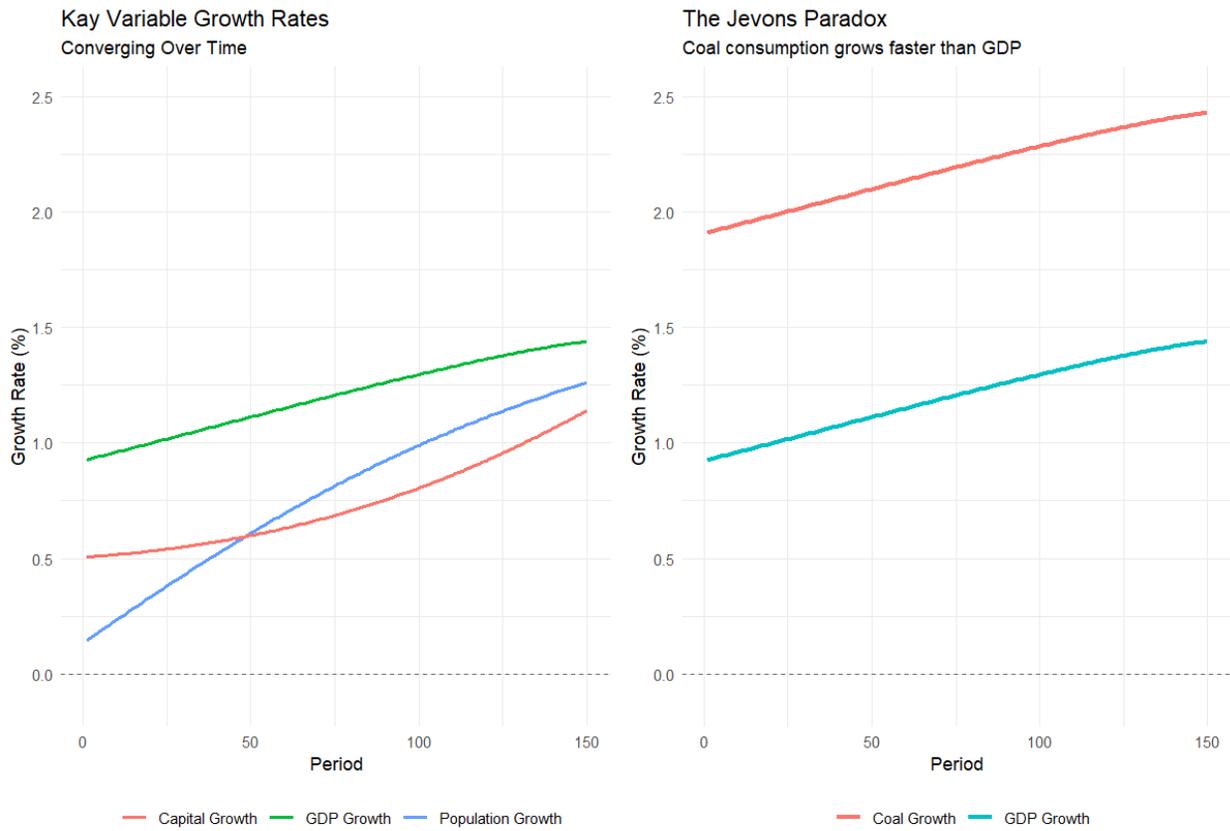
Figure 2: Baseline Growth Rate Dynamics

rising from 1.9% to 2.4% annually while GDP growth rises from 0.9% to 1.4%. The gap widens over time as both Jevons effects and Malthusian population pressure intensify. This means that, despite achieving 0.8% annual efficiency improvements through better steam engines, improved metallurgy, and superior mining techniques, total coal consumption accelerates. The behavioral response ($\sigma = 2.21$) overwhelms technical savings.

5.6 Sensitivity Analysis

Figure 3 presents systematic sensitivity analysis, varying six key parameters $\pm 20\%$ around baseline values. Each panel displays GDP per capita trajectories over 150 periods under different parameter specifications. The analysis reveals differential parameter importance while demonstrating overall model robustness.
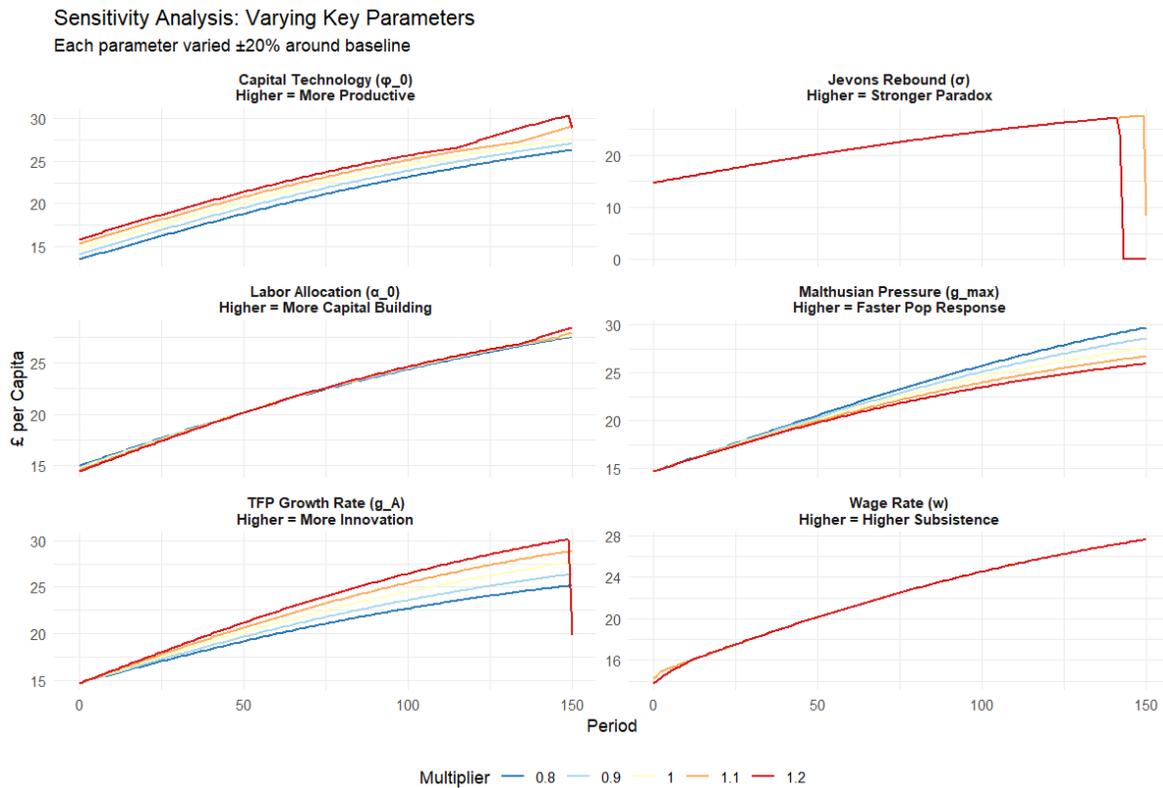

Figure 3: Sensitivity Analysis

Growth drivers show modest variation. Capital technology ϕ_0 and TFP growth g_A,

which are the model's two primary growth mechanisms, both generate upward trajectories with terminal values spanning approximately £25–30 by period 150. Higher capital-building productivity (top-left panel) produces modest gains, with the 1.2× scenario reaching £30.2 versus £26.8 in the 0.8× case. Similarly, TFP growth variation (lower-left panel) yields £25–30 terminal outcomes. These parameters affect growth rates and absolute scales but operate within the classical trap's fundamental constraints.

Malthusian pressure creates meaningful divergence. The population response parameter g_{max} (middle-right panel) generates substantial spread in outcomes. Lower Malthusian pressure (0.8× multiplier, allowing population to respond more slowly to income gains) permits GDP per capita reaching £30, while stronger pressure (1.2× multiplier) holds outcomes near £26. This 15% variation confirms population dynamics' role in determining whether productivity gains translate into per capita welfare improvements.

Labor allocation and wage rate show limited sensitivity. Initial labor allocation to capital building α_0 (middle-left panel) and the wage rate w (bottom-right panel) exhibit relatively tight trajectories. Terminal outcomes vary by less than £2–3 across the full ±20% range for these parameters, indicating that factor allocation decisions and subsistence wage levels matter less than fundamental growth and constraint parameters.

Jevons rebound generates catastrophic nonlinearity. The behavioral elasticity parameter σ (top-right panel) displays dramatically different dynamics. Trajectories proceed similarly through period 100–120 regardless of parameter value. However, stronger rebound effects ($\sigma = 1.2×$ baseline, shown in red) trigger complete system collapse around period 130–140, with GDP per capita falling to near zero as resource exhaustion strikes. Even modest increases above baseline (orange line, 1.1×) show terminal outcomes declining to £10, half the baseline's £27.5. This extreme sensitivity reveals why efficiency improvements can prove catastrophic under classical constraints with behavioral responses that overwhelm technical savings accelerate resource depletion beyond sustainable rates, causing economic collapse rather than gradual stagnation.

Across most reasonable parameter ranges, terminal GDP per capita clusters within £25–30 range, demonstrating the model's fundamental robustness to specification uncertainty. The exception is Jevons rebound which highlights the critical distinction between parameters affecting growth rates versus those determining sustainability. Capital technology, TFP growth, labor allocation, and wage parameters influence transition dynamics and absolute scales but yield qualitatively similar outcomes. Only Malthusian pressure and Jevons rebound fundamentally alter whether the economy sustains modest growth or collapses entirely, confirming these mechanisms as the likely binding constraints within the classical trap.

6 Counterfactual Experiments

In this section I investigate what changes were necessary and sufficient for escaping the classical trap. The experiments compare GDP per capita multiples (ratio of period 150 to period 0 levels) across scenarios, with the baseline achieving 1.88× growth matching England's 1700–1850 experience.

6.1 Experimental Design

The baseline scenario maintains all classical constraints active with modest TFP growth (0.63% per period). I implement ten counterfactual scenarios systematically testing single interventions (1–6), partial combinations (7–8), and comprehensive packages (9–10). Each experiment runs for 150 periods matching the 1700–1850 timeline.

Single interventions test breaking individual constraints:

1. **Demographic transition**: Replace Malthusian population dynamics with fertility declining as income rises (elasticity = 0.6) plus a time trend capturing cultural and institutional changes

2. **Technology acceleration**: Boost capital-building technology by 50% (ϕ multiplier = 1.5×)

3. **TFP acceleration**: Accelerate total factor productivity growth from 0.63% to 1.13% per period (adding 0.5% exogenous growth)

4. **Resource discovery**: Inject 50,000 tonnes of coal equivalent per period (representing new coal fields and fossil fuel transitions)

5. **Land improvement**: Reduce quality degradation parameter by 30% (ξ from 0.10 to 0.07)

6. **Resource efficiency boost**: Increase resource efficiency improvement rate by 30% (1.3× multiplier on ϵ)

Combined interventions test necessity of coordination:

7. **Demo + Tech**: Demographic transition plus technology acceleration

8. **All EXCEPT Demographic transition**: Technology boost (1.5×), TFP acceleration (+0.5%), resource discovery (50k tonnes/period), land improvement (3%), efficiency boost (1.3×)

9. **Moderate escape**: All interventions with modest parameters (demographic transition, tech 1.5×, TFP +0.5%, resources 100k tonnes/period, land 3%, efficiency 1.3×)

10. **Strong escape**: All interventions with aggressive parameters (demographic transition, tech 2.0×, TFP +1.0%, resources 500k tonnes/period, land 5%, efficiency 1.8×)

6.2 Single Interventions: Most Fail or Collapse

Table 3 presents results for single interventions. The findings demonstrate that breaking individual constraints proves largely ineffective or catastrophic.

Table 3: Single Intervention Results

Scenario	GDP p.c. Multiple	vs. Baseline
Baseline (all constraints active)	1.88×	—
1. Demo Transition Only	8.24×	+338%
2. Tech Acceleration Only	**0.00× (collapse)**	-100%
3. TFP Acceleration Only	**0.00× (collapse)**	-100%
4. Resource Discovery Only	1.88×	0%
5. Land Improvement Only	1.88×	0%
6. Efficiency Boost Only	**0.00× (collapse)**	-100%

Demographic transition alone yields 8.24× per capita growth (+338% vs. baseline), by far the best single intervention. Breaking the Malthusian constraint permits sustained accumulation of productivity gains. Final GDP per capita reaches £121 compared to baseline's £28, while population stabilizes at 1.25 million rather than exploding to 16.6 million. However, the level falls short of that actually achieved in the long run, indicating demographic transition, while necessary, proved insufficient without complementary changes.

Technology acceleration alone causes complete collapse. Boosting capital-building productivity (ϕ multiplier $= 1.5\times$) without demographic transition accelerates capital accumulation and output growth initially. Higher output triggers Malthusian population explosion, with population reaching 19.8 million. More economic activity consumes resources faster, amplified by Jevons paradox as efficiency improvements backfire. The system exhausts resources by period 130 and GDP per capita falls to zero. Technological progress without population control proves catastrophic.

TFP acceleration alone similarly collapses. Accelerating productivity growth from 0.63% to 1.13% per period raises output substantially in early periods. However, Malthusian population response overwhelms gains. Population explodes, resource consumption accelerates through Jevons effects, and the economy depletes its resource stock before period 150. This demonstrates that sustained productivity growth, the defining feature of modern growth models, cannot generate escape without breaking the population constraint.

Resource discovery alone produces zero improvement (1.88× identical to base-

line). Injecting 50,000 tonnes of coal per period, roughly matching baseline consumption at period 150 maintains final per capita GDP at £28.

Land improvement yields negligible gains (1.88×, functionally zero improvement). Reducing quality degradation by 30% slightly relaxes the Ricardian constraint but cannot overcome Malthus. The result suggests that agricultural improvements alone, even substantial ones, were insufficient for escape. England's agricultural revolution of crop rotation, selective breeding, and improved implements provided necessary conditions but not sufficient ones.

Efficiency improvement alone causes collapse. Increasing the efficiency improvement rate by 30% (ϵ multiplier $= 1.3\times$) strengthens the Jevons paradox and the behavioral response to cheaper resources overwhelms technical savings by larger margins. Faster apparent progress in coal utilization triggers more rapid consumption growth. The economy depletes resources and collapses.

These results show that the Malthusian constraint dominates all others. Single interventions that fail to address population dynamics either produce no improvement (resources, land) or accelerate system collapse (technology, TFP, efficiency). Only breaking the Malthusian link generates sustained per capita gains, but even this is modest compared to the magnitude of historical escape.

6.3 Combined Interventions: Coordination and Complementarity

Table 4 examines combined interventions, revealing the critical importance of coordination across multiple constraints.

Demographic transition plus technology acceleration achieves 8.48× growth (+351%), marginally exceeding demographic transition alone (8.24×). The combination demonstrates complementarity as breaking the population constraint permits technology gains to accumulate rather than being absorbed. However, 8.48× remains well short of long run experience post 1850, indicating additional changes were necessary.

Table 4: Combined Intervention Results

Scenario	GDP p.c. Multiple	vs. Baseline
Baseline (all constraints active)	1.88×	—
7. Demo + Tech	8.48×	+351%
8. All EXCEPT Demo	2.57×	+37%
9. Moderate Escape	17.83×	+849%
10. Strong Escape	42.78×	+2,178%

The All EXCEPT Demographic transition provides critical insight as, despite receiving five simultaneous interventions, the outcome barely exceeds the baseline's 1.88×. Population explodes to 53.2 million (versus baseline's 16.6 million), absorbing nearly all productivity gains. Per capita GDP reaches merely £44 versus baseline's £28. This result demonstrates that demographic transition is not merely helpful but critically constraining. Massive interventions across technology, productivity, resources, land, and efficiency prove largely futile without controlling population growth.

Moderate escape (all interventions, modest parameters) achieves 17.83× growth (+849% vs baseline). The comprehensive package enables sustained growth by simultaneously relaxing all constraints: demographic transition stabilizes population at 1.29 million, TFP acceleration provides cumulative productivity gains beyond Ricardo's learning-by-doing, resource discovery expands limits faster than consumption growth, land improvement reduces degradation, and technology boosts short-run productivity. Per capita GDP reaches £308. The result demonstrates that coordinated packages can generate substantial improvements even with moderate individual interventions.

Strong escape achieves 42.78× growth. This scenario requires aggressive parameters across all dimensions: demographic transition with fertility collapse, technology boost, TFP acceleration, massive resource expansion, land improvement, and substantial efficiency gains. Per capita GDP reaches £830, representing a transformation from pre-industrial subsistence to modern prosperity.

6.4 Trajectories and System Dynamics

Figure 4 plots GDP per capita trajectories for key scenarios. Strong Escape exhibits near exponential growth, reaching £830 per capita by period 150. Moderate escape shows steady acceleration to £308. Demo+Tech and Demo-Only grow more modestly to £147 and £121 respectively. Baseline remains trapped near £28. The All EXCEPT Demo trajectory rises initially as technological improvements take effect, reaching £60 by period 100, then stagnates as population growth overwhelms gains, finishing at £44. The collapse scenarios (such as TFP Only) initially rise then crash dramatically around period 120–130 when resource exhaustion strikes, falling to zero.
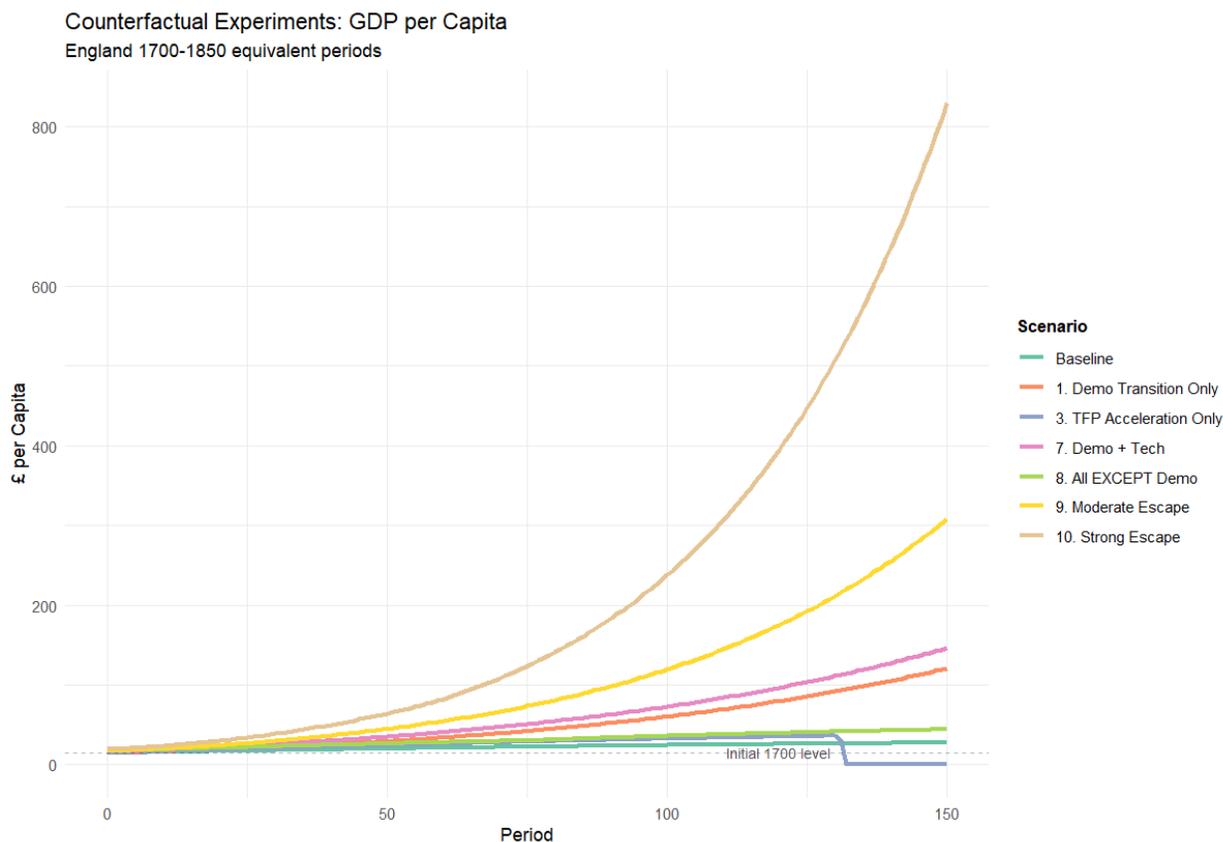

Figure 4: Counterfactual Experiments: GDP per Capita Trajectories.

The resource constraint dynamics (Figure 5) reveal the mechanism of collapse. Baseline gradually depletes the 3,281 million tonne initial stock, consuming 2,300 million tonnes

by period 150 with 900 million remaining. Collapse scenarios deplete stocks catastrophically, hitting zero around period 130 when consumption explodes due to combined Malthusian population growth and Jevons efficiency paradox. Moderate escape maintains resource abundance by discovering additional resources, while strong escape dramatically expands resources, representing the scale of fossil fuel transitions (coal to oil to natural gas) that eventually characterized industrial modernity.
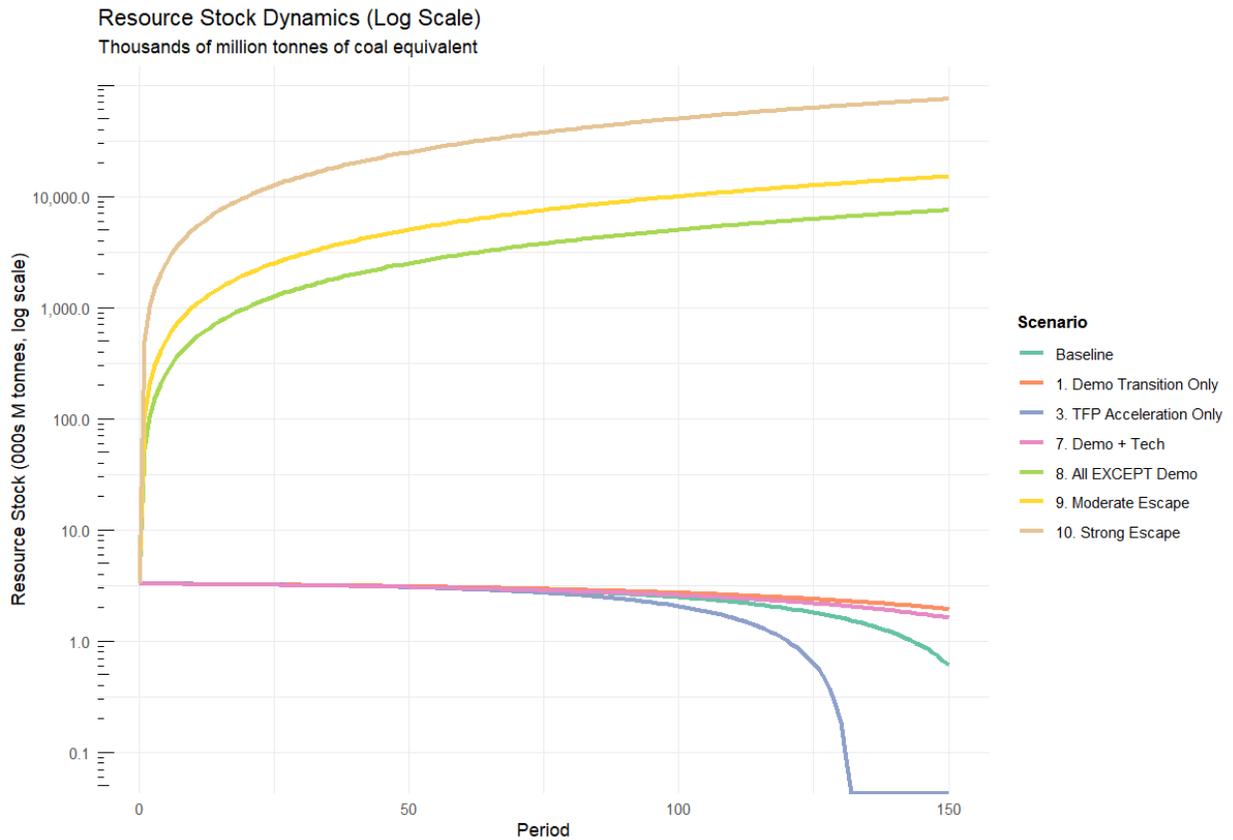

Figure 5: Resource Stock Dynamics Across Scenarios (Log Scale).

7 Conclusion

This paper provides the first formal synthesis of Malthus's population dynamics, Ricardo's land degradation and capital accumulation, and Jevons's efficiency paradox. The unified MRJ model achieves exceptional empirical fit, matching England's 1700–1850 experience across population, GDP, land, coal, capital, and wages with less than 1% error. The model captures the central historical puzzle where dramatic technological transformation coincided with modest living standards improvement. This tight historical fit validates that classical economists identified genuine structural features of pre-industrial economies. Their pessimism reflected not analytical failure but accurate diagnosis of multiple binding constraints operating simultaneously in the transition economy they inhabited.

The counterfactual experiments yield three core insights. First, demographic and resource constraints bind far more tightly than others. Technology acceleration, TFP growth, and efficiency improvements all trigger system collapse without demographic transition, as unchecked population growth exhausts resources through Jevons-amplified resource consumption. Second, demographic transition proves necessary but insufficient as breaking the Malthusian constraint alone achieves only 8.24× growth, far short of that observed post-1850. Third, and most strikingly, implementing *every* improvement except demographic transition produces barely any per capita improvement (2.57× versus baseline 1.88×) as population explodes to absorb all gains. Sustained escape requires coordinated breaks across all constraints simultaneously.

These findings help explain why England's escape was historically unusual. Improving living standards required an extraordinary confluence of fertility decline beginning after 1870, sustained productivity acceleration through knowledge accumulation and institutional change, fossil fuel transitions expanding resource access, and agricultural improvements relaxing land constraints. Further, all of these had to occur almost simultaneously. This explains both the rarity of successful industrialization and why classical economists, observing genuinely binding constraints, reached pessimistic conclusions that proved correct

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until historically contingent changes overwhelmed established mechanisms. The results also speak to contemporary developing economies exhibiting similar constraint structures with ongoing technological change and capital accumulation that fails to translate into sustained per capita growth. The classical trap's logic of population pressure absorbing productivity gains, efficiency improvements causing rebound effects and resource constraints binding remains relevant where coordinated breaks have not occurred.

By taking Malthus, Ricardo, and Jevons seriously as constraint theorists rather than dismissing them as outdated pessimists, this paper demonstrates that classical economics, properly formalized, yields a coherent framework for understanding both pre-industrial stagnation and the conditions for escape. The pessimism of the classical economists reflected genuine mechanisms that bound the economy for millennia. Their refutation required simultaneous revolutions across every dimension of economic life; a transformation so extraordinary that its rarity, not its occurrence, forms the basis for economic understanding.

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A   Mathematical Derivations

This appendix provides formal proofs for the regularities stated in the body of the paper. All proofs assume zero exogenous TFP growth ($g_A = 0$), corresponding to the pure classical framework. This assumption is appropriate for three reasons. First, it reflects historical fidelity: Malthus (1798), Ricardo (1817, 1821), and Jevons (1865) developed their theories without access to concepts of sustained economy-wide productivity growth, which entered economic theory only with Solow (1956). Second, establishing results for $g_A = 0$ demonstrates that classical pessimism followed logically from classical premises, that is, their predictions of stagnation were not analytical errors but correct deductions from their assumptions. Third, the calibration shows that introducing historically-observed TFP growth ($g_A = 0.63\%$ per annum) does not overturn these results as the model still reproduces England's 1700–1850 experience with per capita income remaining close to subsistence despite continuous technological progress. The proofs thus establish the theoretical benchmark against which the quantitative model's behavior can be understood.

A.1   Proposition 1: Classical Stationary State

We prove the model converges to a quasi-stationary state where per capita income equals subsistence and growth rates approach zero.

Step 1: Land growth rate approaches zero.

Physical land grows arithmetically:

$$Land_{phys,t} = Land_0 + g_{land} \cdot t$$

The absolute growth rate is constant (g_{land}) but the percentage growth rate declines:

$$\frac{Land_{phys,t+1} - Land_{phys,t}}{Land_{phys,t}} = \frac{g_{land}}{Land_0 + g_{land} \cdot t} \to 0 \text{ as } t \to \infty$$

Effective land growth declines even faster due to quality degradation ($\xi > 0$):

$$Land_{eff,t} = Land_0 \left(\frac{Land_{phys,t}}{Land_0} \right)^{1-\xi}$$

Taking logs and differentiating with respect to time:

$$\frac{d \ln Land_{eff}}{dt} = (1 - \xi) \frac{d \ln Land_{phys}}{dt}$$

Since $1 - \xi < 1$ and $d \ln Land_{phys}/dt \to 0$, we have $d \ln Land_{eff}/dt \to 0$ at a faster rate.

Step 2: Malthusian dynamics force convergence to subsistence.

The population adjustment mechanism creates negative feedback:

- If $y_t > \bar{y}$: $g_{pop,t} > 0$, so $N_{t+1} > N_t$. With circulating capital growing more slowly than population (see Step 5), per capita income $y_{t+1} = K_{circ,t+1}/N_{t+1}$ falls.

- If $y_t < \bar{y}$: $g_{pop,t} < 0$, so $N_{t+1} < N_t$. Per capita income rises.

This feedback mechanism is stable and forces convergence: $y_t \to \bar{y}$ as $t \to \infty$.

Step 3: Equal growth rates for output and population.

In steady state with $y_t = \bar{y}$:

$$\frac{Y_t}{N_t} = \bar{y} \implies \frac{Y_{t+1}}{N_{t+1}} = \bar{y}$$

Therefore:

$$\frac{Y_{t+1}/Y_t}{N_{t+1}/N_t} = 1 \implies g_{Y,t} = g_{N,t}$$

Step 4: Growth rates converge to zero.

From the production function (equation 9):

$$Y_t = A \cdot (\phi_t K_{fixed,t})^\kappa \cdot L_{cons,t}^\lambda \cdot Land_{eff,t}^\mu$$

With $\kappa + \lambda + \mu \leq 1$ (decreasing returns) and $Land_{eff,t}$ growth approaching zero (Step 1), output growth is bounded by effective land growth. As percentage land growth approaches zero, so must output growth.

Combined with Step 3: $g_{Y,t} \to 0 \implies g_{N,t} \to 0$.

Step 5: Circulating capital tracks population.

The employment constraint (equation 4) requires $L_t \approx N_t$ for full employment, which necessitates:

$$K_{circ,t} \approx w \cdot N_t$$

Therefore circulating capital must grow at the population rate to maintain employment. In steady state with $g_{N,t} \to 0$, circulating capital growth also approaches zero.

A.2 Proof of Proposition 2: Technology Neutrality

We prove that improvements in capital-building technology (ϕ_t) or technical resource efficiency ($\eta_{technical,t}$) do not raise steady-state per capita income.

This result follows from the Malthusian convergence established in Proposition 1. Since $y_t \to \bar{y}$ regardless of technology levels, technology cannot affect steady-state per capita welfare. We trace the mechanism for each technology type.

Capital-building technology ($\phi_t \uparrow$):

Higher ϕ_t accelerates capital accumulation:

$$\Delta K_{fixed,t} = \phi_t \cdot L_{capital,t}^{\eta_{capital}}$$

Increased capital raises output through the production function, temporarily elevating per capita income above subsistence ($y_t > \bar{y}$). The Malthusian response follows: population growth accelerates, dividing output among more people until y_t returns to $\bar{y}$. In the new steady state, K_{fixed} and N are both higher, but $y = Y/N = \bar{y}$ as before.

Technical efficiency ($\eta_{technical,t} \downarrow$):

Improved technical efficiency reduces resource requirements per unit output. However, as established in Proposition 3, actual resource intensity η_t rises when $\sigma > 1$ due to behavioral responses:

$$\eta_t = \eta_{technical,t}^{1-\sigma} \cdot \eta_0^\sigma \implies \frac{\dot{\eta}_t}{\eta_t} = (\sigma - 1)\epsilon > 0$$

Nevertheless, comparing economies with different technical efficiency levels, greater efficiency supports higher output from given resources. This higher output temporarily raises $y_t > \bar{y}$, triggering population expansion until y_t returns to $\bar{y}$. Technical efficiency determines how large a population can be supported, but not how well that population lives.

In both cases, technology affects the scale of population and capital but not per capita welfare, which remains anchored at $\bar{y}$.

A.3 Proof of Proposition 3: Jevons Resource Paradox

We prove that resource consumption growth exceeds output growth when $\sigma > 1$, causing accelerating resource depletion despite continuous efficiency improvements.

Resource consumption is defined as:

$$R_{consumed,t} = \eta_t \cdot Y_t \tag{26}$$

Step 1: Derive the growth rate of actual resource intensity.

Technical efficiency improves exponentially:

$$\eta_{technical,t} = \eta_0(1 - \epsilon)^t$$

Behavioral response determines actual intensity:

$$\eta_t = \eta_{technical,t} \cdot \left(\frac{\eta_0}{\eta_{technical,t}}\right)^\sigma = \eta_{technical,t}^{1-\sigma} \cdot \eta_0^\sigma$$

Taking logarithms and differentiating with respect to time:

$$\frac{d \ln \eta_t}{dt} = (1 - \sigma)\frac{d \ln \eta_{technical,t}}{dt}$$

Since $\frac{d \ln \eta_{technical,t}}{dt} = \ln(1 - \epsilon) \approx -\epsilon$ for small ϵ:

$$\frac{\dot{\eta}_t}{\eta_t} = (1 - \sigma)(-\epsilon) = (\sigma - 1)\epsilon$$

For $\sigma > 1$, the growth rate of actual resource intensity is *positive*: $\dot{\eta}_t/\eta_t > 0$. Despite continuous technical efficiency improvements, behavioral responses cause actual resource intensity to rise.

Step 2: Derive the growth rate of resource consumption.

The growth rate of resource consumption equals the sum of intensity and output growth rates:

$$\frac{\dot{R}_{consumed}}{R_{consumed}} = \frac{\dot{\eta}}{\eta} + \frac{\dot{Y}}{Y} = (\sigma - 1)\epsilon + g_Y$$

where $g_Y = \dot{Y}/Y$ is output growth.

Step 3: Compare resource consumption growth to output growth.

The difference between resource consumption growth and output growth is:

$$\frac{\dot{R}_{consumed}}{R_{consumed}} - \frac{\dot{Y}}{Y} = (\sigma - 1)\epsilon$$

For $\sigma > 1$ and $\epsilon > 0$, this difference is strictly positive. Resource consumption grows faster than output in every period.